

Changing representations and practices in school mathematics: the case of Modern Math in Portugal

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Abstract

The curricular movement known as Modern Mathematics aimed at the transformation of representations and practices in school mathematics. Its study provides us with ways of understanding how these changes came about. The purpose of this paper is to contribute to the understanding of the ways in which representations of school mathematics gradually were influenced by ideas from the Modern Mathematics movement, how these new ideas merged into local educational traditions, and how they were transformed into meaningful practice. This work is centred on the Portuguese context from the middle 1950s to the middle 1960s, and builds on Chervel's notion of school culture and Gruzinski's discussion of connected histories.

Introduction

This paper intends to contribute to the understanding of changes in the school culture, especially as they relate to teaching and learning of mathematics. The notion that school disciplines acquired a considerable degree of autonomy, in relation both to other school areas and to scientific disciplines, was first put forward by André Chervel (1988). Shifting the focus of research — either from schools as sociological global entities or from disciplines as vulgarizations of scientific knowledge — permitted educators to pose new research problems that valued the importance of school disciplines (including their associated syllabi) as specific and autonomous creations of the school culture.

The Modern Mathematics movement¹ spread especially from the 1960s through many countries attempting major changes in content and methods for teaching mathematics. Its specificity — as a school *mathematics* reform — and its broadness — either geographically or in terms of the targeted grade levels — provide an exceptional field for comparative research into the ways in which such reform came about and how math school culture in distinct school systems was affected.

School culture was specified by Dominique Julia (1995) as a set of *norms* that define knowledge to teach and conducts to inculcate, and as a set of *practices* allowing the transmission of this knowledge and the incorporation of these behaviors. This paper intends to understand how representations (here identified as Julia's norms) of school mathematics gradually were influenced by

¹ In this text, this movement will be referred also as new math. Occasionally for the sake of clarity, the term “classic” will be opposed to the “modern” ideas.

ideas from the Modern Mathematics movement, how these new ideas merged into local educational traditions, and how they were transformed into meaningful practice. This paper is a partial account of on-going comparative work by two groups of researchers in Brazil and Portugal².

A broad description of the movement can be found in Moon (1986). A chronology of the national and international events leading to the reform was established in Matos (1989) and detailed partial analyses can be found in Almeida (2007) and Candeias (2008). This paper is based on data collected through documentary research. Sources include articles in professional journals and magazines, official documents, gray documents (teachers' reports, minutes from teachers' councils, students' workbooks, and the like), and textbooks. Interviews with teachers and former students were also conducted.

The context

After a reform in 1947, the centralized Portuguese school system consisted of a primary cycle, followed by parallel branches for the secondary schools: the *liceus* that after 7 years of schooling aimed at matriculation at the universities, and the *técnicas*, technical schools aimed at the educations of workers at several skill levels (Figure 1).

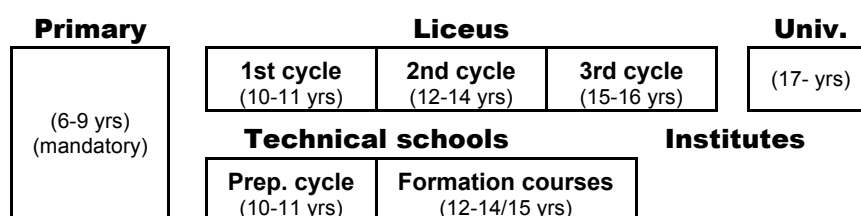


Figure 1. Portuguese school systems, 1947.

In the beginning of the 1950s, Portuguese society was adjusting to the post-war era. Although political leadership remained authoritarian, some forces inside and outside the establishment aimed at changes. A drive towards strengthening industry, furthering economic integration with other countries, and participating in the international (economical, political, and military) organizations emerging in the Western countries took place, led by the so-called “developmentalists” (“desenvolvimentalistas”) inside the regime. In education, from 1956 and especially from 1962, several adjustments were put forward under their initiative: mandatory schooling was expanded, teacher certification increased in scope, and programs at the universities were updated.

² Project *Modern mathematics in Brazil and Portugal: a comparative history study*, jointly supported by the Portuguese FCT — *Fundação para a Ciência e a Tecnologia* and the Brazilian CAPES — *Coordenação de Aperfeiçoamento de Pessoal de Nível Superior*.

The reform of Modern Mathematics

The reform of Modern Mathematics occurred in this context. The events can be divided into three intertwined periods: *the beginnings*, from 1957 until 1963, in which the flow of new ideas can be detected; *experimentation*, from 1963 to 1968, during which the new ideas were implemented in classrooms; and *dissemination*, from 1968, that saw the gradual generalization of the reform to all students. For some branches of the educational system, these three periods overlap to some extent.

The first period, *beginnings*, highlights the concept of *connected history* (Gruzinski, 2003), developed by cultural historians, that has been used to understand how communities articulate their representations and practices in connection with other communities. Going beyond both a mirror perspective — in which one culture tries to emulate another — and a power perspective — interpreting cultural influences as either acts of emulation or resistance, the influence of one culture by another is seen as acts of *hybridism* (*métissage*), in which external appropriations are not seen as imitations (or distortions) from the original, but as producers of a new originality. In this model, a key role on the circulation of ideas and practices has been attributed to *mediators*, persons that travel between societies and cultures.

The role of mediators was prominent in the expansion of the new ideas in Portugal. The mathematician José Sebastião e Silva (1914-1972) became the best known leader of the movement in Portugal. He had close contact with Federigo Enriques, Guido Castelnuovo, and Emma Castelnuovo, among others, when studying in Rome from 1943 to 1946. He was a regular presence at international meetings, including the 1952 general assembly of the International Mathematical Union in Rome that re-instated the International Commission on Mathematical Instruction (ICMI) (Matos, 1989).

In 1957, following participation in the eleventh meeting of the *Commission Internationale pour l'Étude et l'Amélioration de l'Enseignement des Mathématiques* (CIEAEM), the new curricular ideas became known to a wide audience of educators. A delegation to that meeting, appointed by the Portuguese government, was composed of José Sebastião e Silva; José Calado, Jaime Furtado Leote, who were both teachers in a *Licen*, and Santos Heitor, teacher at an *Escola Técnica* (Silva, 1957). These three educators were teacher trainers with responsibilities for the initial formation of teachers (*metodólogos*). Sebastião e Silva and José Calado were members of the Portuguese Sub-Commission of ICMI³, and Calado, Heitor, and Silva were also textbook authors.

There are two reasons to consider this trip a major event. First, as Heitor (1958) puts it: “it was a first presence at meetings, that have been occurring for years” (p. 270)⁴. Following internal policies, public servants were not allowed to

³ Appointed in 1955.

⁴ All quotations were translated by the author, J. M.

travel to foreign countries and thus, for over 30 years, there had been no participation by Portuguese teachers in international events. Second, all four participants, acting as mediators, published articles about this experience. At least until 1963, others mentioned this congress as well. Their reporting of this event is therefore very important for the understanding of the initial representations of the Modern Mathematics movement in Portugal.

A few months after the CIEAEM meeting, a major conference at *Licéu Pedro Nunes* in Lisbon brought together many influential educators, among them principals, administrators, and the Minister of Education (a former teacher of mathematics) himself (Calado, 1958). The presenter, José Calado, started by outlining the concerns of ICMI. Then, he explained the “spirit of modern mathematics” (p. 92), and outlined a programme for action: a curricular revision with more time in class spent on mathematics, and algebra and logic courses for teachers.

After one course on logic conducted by Sebastião e Silva in 1958, no other initiatives were undertaken⁵. The subject of the new international tendencies, however, continued to be discussed (Matos, 2006), either in professional journals or in *estágios* (two-year teacher in-service training programs).

It is usually agreed that the second period, the *experimentation* of the new math curricula in Portugal, began in 1963. Actually, from 1961 the new ideas started to be experimented with in the classes at a private primary school owned by João Nabais (Candeias, 2008).

In the middle of 1963, a Commission for the reform of the upper cycle of *licéus*, composed of Sebastião e Silva and three teacher trainers — Jaime Leote, Manuel Augusto da Silva, and Augusto Lopes, later joined by an Inspector⁶, was appointed. By the end of 1963, Modern Mathematics was taught to three special classes in the 3rd cycle of *licéus*. In the following years, these classes gradually spread and, in 1964, Sebastião e Silva wrote a textbook for them. This experiment was supported by the Organization for Economic Co-operation and Development (OECD) and in 1963 members of the Commission attended the Athens conference, which focused on the on-going reform in several OECD countries (Almeida, 2007; Matos, 1989).

Other branches of the Portuguese school system also experimented with the new ideas. First, *Telescola* — a national network of schools complemented by classes on television — broadcast a series of math lessons by Augusto Lopes from 1964, and later, from 1968, introduced Modern Mathematics at the technical schools (Matos, Novaes, & Gabriel, 2009). In this last case, the reform was supported by what can be considered the first Portuguese journal on mathematics education, *Folha Informativa*.

⁵ The conjecture that this was due to political instability both internal and external to the regime still has to be confirmed.

⁶ Apparently José Carneiro da Silva.

The last period, *dissemination*, started in 1968, when the first cycles of *licens* and technical schools were merged into a new autonomous cycle for 10/11 year-old children that included Modern Mathematics in its program (Wiliewsky & Matos, 2009). From 1968 until 1974, programs were sequentially modified to incorporate the new ideas as these pupils progressed through the cycles. After gradual changes, the new math programs were replaced in early 1990. Primary schools were left out of these programs until 1974.

Changes in representations

Modern math acted on the representations component of school mathematics culture across four dimensions: a) on the nature of learning; b) on the school mathematical content; c) on the methodologies for teaching mathematics, and d) on the social role of mathematics.

The first dimension — changes in the way learning is perceived — is fairly homogenous across all data. “Learning elementary mathematics is within the reach of all our pupils,” stated the teacher trainer Jaime Leote (1958, p. 37). His “conviction” is based on personal experience and on the work of Piaget: “it is intelligence’s own activity that generates mathematical relations” (p. 37), or, as another teacher trainer, Augusto Lopes, stated, “mathematical knowledge is born and develops by interiorizing concrete actions and the organization of operatory schemas” (1960, p. 634). More explicitly,

in children, intelligence is essentially the coordinator of motor or sensory-motor actions, organizing them in schemas. Actions are interiorized then in operations, that is, intelligence spontaneously organizes operatory structures that correspond to the fundamental structures of modern math. (Leote, 1958, p. 38)

Quotations like this can be found throughout the entire period under scrutiny, and this correspondence between cognitive and mathematical structures is presented as a major finding in psychology that legitimated crucial choices, both of content and of methods.

There are other references to the ways in which learning is perceived. In contemporary terms, we would name them as following a constructivist perspective. For example, quotations such as “the act of learning must be an act of creation and not a merely receptive act” (Calado, 1958, p. 91) can be found in many documents. This perspective expresses a vision put forward by the proponents of the *New School* early in the XXth century (influenced by Edouard Claparède and Adolphe Ferrière, for example) (Fernandes, 1978) that was commonly studied in Portuguese teacher education at least from the 1930s and so antedates Modern Mathematics.

Changes in school mathematics content — the second dimension of change in school mathematics culture — gradually built from early opinion articles, to programs, and to textbooks. From 1957, many authors emphasized the terms “relations”, “structures”, or “operations”, shifting the center of mathematics teaching: “the essential objective is the study of algebraic structures” (Calado,

1958, p. 93). Moreover, it was expected that the Modern Mathematics approach clarified, simplified, and unified school mathematics (Heitor, 1967).

Many of the teachers involved in teacher education, most of them also textbook authors, published similar opinions. However, there are indications that these positions were not unanimous. At least one prominent teacher and textbook author, Francisco Gonçalves, questioned the trends on the grounds that 50% of the students failed examinations in the 2nd and 3rd cycles of *licens*, in spite of the “successive simplifications that we introduce in our courses” (Gonçalves, 1961, pp. 546-7)⁷.

In the field of education it is necessary to pragmatically put aside matters and methods that were already old 50 years ago, but at the same time not show willingness to implement every fashion coming from Paris, even if it comes with the guarantee of the fascinating name of Nicolas Bourbaki (Gonçalves, 1961, p. 548).

In 1962, when he was already preparing the introduction of Modern Mathematics in the upper classes of *licens*, Sebastião e Silva published an article discussing the options for the mathematical content of the experiment that would start in 1963. He argued that

these innovations must be performed with extreme prudence and the most skilled pedagogical touch, if we do not wish to create in students an invincible repulsion for mathematics or lead them to the acquisition of an empty formalism, that completely sterilizes. (Silva, 1962, p. 25)

Consequently, he proposed that the programs of the first five years of *licens* remain classic in their general lines, but strongly influenced by the modern spirit from the first year. Ideas, methods, and language of modern mathematics will be gradually introduced as classic topics occur. (ibid., pp. 25-6)

During the last two years of *licens*, however,

[the program] should move to a concentrated, systematic study, even at a still elementary level, of several topics of modern mathematics, either pure (mathematical logic, set theory, abstract algebra) or applied (probabilities and statistics). (ibid., p. 26)

By 1964, Sebastião e Silva produced a *Compendium of mathematics* in four volumes for use in the experimental classes of the two last years of *licens* (Silva, 1964/1975) together with a *Guide for teachers* in three volumes (Silva, 1975). These books outlined a coherent approach to Modern Mathematics for students intending to pursue their education at universities, and they blended the “classical” with the “modern” vision of mathematics. They are still cited as a reference in contemporary programs. Table 1 shows the percentage of pages devoted to key topics in the *Compendium*.

⁷ To fully appreciate the dimension of these rates of failure, one must take into account that, at the time, students’ population of *licens* was a very small percentage of the youth population.

Table 1. Percentage of pages in *Compendium* by year and topic.

Year	Topic	pp.	%
1 st	Logic and set theory	122	25
	Combinatorics	38	8
	Functions	38	8
	Algebra (rings, fields, Boolean algebras)	190	39
	Statistics and probability	98	20
	Total (1 st and 2 nd tome of 1 st volume)	486	100
2 nd	Differential and integral calculus	308	50
	Deductive arithmetic	64	10
	Approximate values	36	6
	Vectors	46	7
	Complex numbers	22	4
	Algebra (affine and linear transformations, matrices)	144	23
	Total (2 nd and 3 rd volumes)	620	100

Note: compiled from Silva (1964/1975).

Algebra and analysis are the core topics, accounting for about one third of the intended course each. Logic and set theory are the next important topics (about one fifth). Sebastião e Silva was also one of the authors of the previous unique textbooks for this grade level. This *Compendium* shows major changes in two domains: first, the “classical” mathematical content, especially analysis, was now permeated by logic and set theory; second, algebra, previously absent, can now be found across most of the topics. Other topics were also added: integral calculus, probability, and statistics.

Mathematical content was also considerably changed by the reform of other branches of the educational system. Perhaps the distinctive characteristic, common to all grade levels, was the adoption of set theory as the appropriate language in which to express mathematics, leading to considerable changes in the ways in which mathematics related to other subjects.

To illustrate this conception, we can compare how two textbooks for the fifth grade presented addition, one of the sections of both the old and the new curriculum. For each of these books this topic is addressed at the very beginning of a chapter. Following the official program that suggested the constant use of geometrical concrete problems, the unique textbook (Ribeiro, 1965) motivated addition by posing a problem involving travelling by train in two journeys and asking what was the total distance of the trip. One of the most successful textbooks introduced after the reform, (Lopes, n.d.⁸, p. 101), introduced this subject as shown in Figure 2.

⁸ The publishing date of this source is not known, but suspected to be one of the first editions, if not the very first one, in 1968.

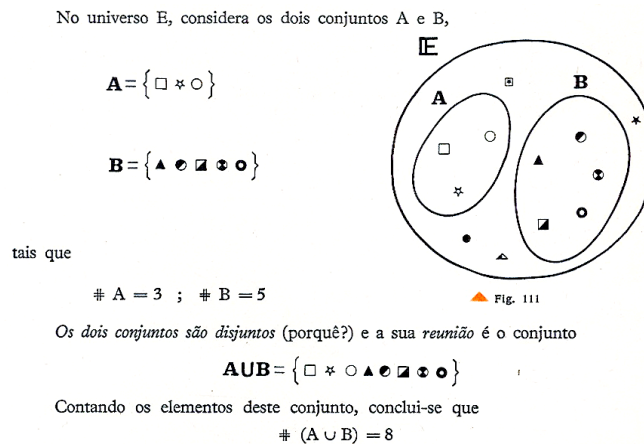


Figure 2. Introduction of addition (Lopes, n.d., p. 101).

Addition was presented in the newer book as the cardinal of the union of two non-intersecting sets of symbols: $\#(A \cup B)$. The connection with common everyday knowledge that legitimated the “classical” approach was replaced by the arbitrariness of a set made of abstract characters. Moreover, the second approach required students to be conversant in the language of sets. The “classic” approach discussed addition in the beginning of the book (after 14% of the pages), whereas the “modern” approach needed almost half the book (45%) to introduce the same topic.

At the time, since cognition was represented as based on a correspondence between mathematical and psychological structures, there were strong reasons to use this approach. Early incorporation of set theory, although more difficult in the short-term, was believed to ensure a better quality of mathematics learning in the long run.

The representations about the methodologies for teaching mathematics — the third dimension of change in the school mathematics culture — were addressed in many documents. The teacher trainer, Jaime Leote, believed that the “heuristic method (...) creates an intellectual environment that facilitates the student to reconstruct a proof by himself, and not to accept it by virtue of the teacher’s authority” (p. 39). Sebastião e Silva (1957), building on Puig Adam and Willy Servais, stated that mathematics teaching

should be based (...) on an intuitive, concrete, heuristic basis. The purpose of this orientation is not just to make teaching more appealing (...); but also, and mainly, to lead students to spontaneously and increasingly elaborate mathematical logical schemes, until their more rational and abstract phase, so that, inversely, they learn to use them in their concrete applications. (Silva, 1957, p. 31)

The qualification “heuristic” is widely used in the documents studied, and it is possible to understand its specific content by analyzing contemporaneous teachers’ reports (Matos & Fischer, in press). For example, Joaquim Preguiça, a

math teacher at a *Licen*, in the mandatory report for 1960 stated that his teaching followed,

when possible, an active experimental method, in which I tried to ensure that children learned through experiences performed in class. The class took, many times, the aspect of a laboratory in which children made drawings, reconstructions, measurements, and weighing and thus were learning the first mathematical notions through intuition and material objects (mathematical models). (Preguiça, 1960, p. 4)

Teacher trainers, from the *licens* (Leote, 1958; Lopes, 1960) and the technical schools (Heitor, 1967), and official recommendations both made significant references to the use of materials, especially geoboards, Cuisenaire rods, and geometrical models. Classroom organization was also occasionally referenced. Leote (1958), for example, proposed that classrooms should be furnished with individual tables that could be rearranged for group work.

As was the case with the representations about learning, valuing these active methodologies was inherited from the perspectives of the *New School* movement from the beginning of the twentieth century (Matos & Fischer, in press). This tradition was blended with materials specifically associated with the new math movement.

A fourth and last detected change in representations concerns the social role of mathematics; that is, the perceived importance of mathematics for society as a whole. These changes can be summarized by the statement given by Sebastião e Silva to a newspaper and chosen as a sub-title: “A modern nation cannot endure without good technicians, good scientists, and ... good teachers” (Enquanto o homem caminha para a Lua ... (2), 1966, p. 1). The same idea was expressed by him nine years earlier:

The modern world will need, on an increasing scale, scientists and technicians with a good mathematical formation. Hence, the need for urgent renewal of, not only mathematical programs, but also methods for teaching this discipline from primary school to the university (Silva, 1957, p. 31).

In a somewhat distinct perspective, Jaime Leote argued for the formative character of mathematics:

In the present society, mathematics cannot lose the classical character of ‘humanities’; it must powerfully contribute to eliminate imprecision, incoherence and arbitrariness, to discipline the spirit and inform it in the light of rigorous reasoning. But we must not forget also that mathematics constitutes, more and more, a prodigious element of work for the technician and for the physicist. (Leote, 1958, p. 38)

And José Calado (1958) proposed that school mathematics should change so that teaching initiated students in the spirit of contemporary science, to avoid the shock with “different notions and in a totally new language” (pp. 91-92) when students continued to the universities.

These two teachers also presented a broader perspective. For example, Leote argued for the importance of mathematics for the West:

Therefore it is especially necessary that young people take interest in the study of mathematics; the Western world lacks elements allowing it to raise technical progress to the especially human and civilized level that characterizes it. (Leote, 1958, p. 38)

José Calado argued that the reform was important because Western nations are striving together to “defend a millenary common civilization” (1958, p. 103).

In sum, these authors, all related to *licens*, asserted the importance of Modern Mathematics to the formation of specialized human labor; namely, technicians and scientists. They did not stress its importance for the education of all students. There are, however, other authors who took a different stance. For example, Francisco Gonçalves (1961), the critic of the Bourbakist approach, acknowledged the importance of scientific preparation for the few, but he mainly argued that general social indicators and international comparisons suggested that the country needed, firstly, to invest in “mathematics for the crowd and not for the elites” (p. 548).

The reform was confronted with a major challenge when it was necessary to introduce it at the technical schools (Matos, Novaes, & Gabriel, 2009). In 1963, when the first experiment started at *licens*, it was thought that Modern Mathematics was not intended for these schools (Almeida, 1970). However, in 1967 its introduction began to be discussed in the Ministry of Education and among math teachers. Until that time, the school mathematics culture of *técnicas* valued instrumental and applied approaches. Mathematics was socially relevant as a tool to be used in the working place. The sum of the objections can be observed in the positions of Álvaro Pereira Gomes, a trainer of teachers in 1968⁹. Stating that he did not oppose Modern Mathematics, he also believed that “Practical Mathematics” must have an important role:

I strongly respect the formative aspect but I also respect very much the application part that must constitute an important role in technical schools. (Pereira, 1968, pp. 13-14)

It was very difficult to understand how Modern Mathematics, valuing broad and abstract structures because of their formative value, could merge into the school culture of *técnicas*, which valued instrumental approaches. In fact, why should those contents be relevant for the future professional lives of the students not geared for the universities? Nonetheless, the reform eventually was applied to these schools and within a few years (from 1975) they merged with the *licens*.

Changes in practices

Research becomes very difficult when investigating past school practices, because usually only indirect evidence can be obtained. In the present case, however, it is possible to determine that representations of Modern

⁹ His positions were recorded in the minutes of a meeting authored by Pereira and published in *Folha Informativa* (Pereira, 1968).

Mathematics affected both the practices of teacher education and of classroom practices.

From 1956 on, the initial in-service formation of secondary teachers, concentrated until then in one *licau*, was expanded, with this new responsibility leading to changes in the school culture of the other two *licaus*¹⁰ (Matos, 2006). There is documentation suggesting that these secondary schools took this task very seriously and tried to be models for excellence in teacher education. At the secondary level, Modern Mathematics ideas were appropriated in this very innovative context.

In the *Licau Normal de Pedro Nunes*, in Lisbon, extensive documentation — usually published in the school journal *Palestra*, and also found in the school's archives — shows that the modern perspectives were discussed in public conferences¹¹, sometimes conducted by the in-service teachers themselves. Records of classroom experimental materials developed by these teachers were published occasionally (Brito, 2006). In the *Licau Normal de D. Manuel II*, at Oporto, there is also evidence that similar events were taking place. Published articles and interviews with some of the participants show that future teachers were producing theoretical elaborations of the innovative trends, and a thorough program for the development of materials for secondary students was put into practice (Anjos, 2008).

At the primary level, there are indications that the Modern Mathematics reform affected practices of the in-service education of teachers, although at a minor scale as compared to the secondary level. From 1962, João Nabais conducted many short-term courses on the use of manipulative materials — essentially Cuisenaire rods, Dienes blocks, and other structured material, for primary teachers (Candeias, 2008).

As for changes in classroom practices, although there is evidence to suggest that these materials were integrated into teaching practices at Nabais's private school, the extent to which they reached other primary schools is not known.

In the case of secondary education, new methods were adopted, at least in some of the *licaus* experimenting with the new programs for the upper secondary cycle. There are photos showing actual experimental classes that were published in a national newspaper (Figure 3) with the legend: "Desks being abolished, allowing for new practical tables supporting teamwork, the mathematics classroom is now really a laboratory in Liceu Pedro Nunes" (Revolução no ensino, 1963, p. 13).

¹⁰ Changes in in-service teacher training at the third *licau* in Coimbra are still unclear. From 1947 until 1956, this was the only school for in-service secondary teacher training.

¹¹ One of the first of these conferences was actually the presentation in 1957 of the new ideas for mathematics teaching by José Calado (1958) in the presence of the Minister of Education, as analyzed previously.



Figure 3. An experimental mathematics classroom
(*Revolução no ensino*, 1963, p. 13).

From the analysis of students' workbooks prepared in the experimental classes (Silva & Valente, 2008), it is possible to infer some routines of the classrooms' daily life. In the same class, for example, the teacher would start with explicit references to the new manual (including the page numbers), propose an activity taken from a well-known book of exercises (whose publication started in the 1940s), move to the "classic" manual to explain some other mathematical aspect, revert to the experimental text, and so on. The implemented curriculum appears as a hybrid, a blend between "new" and "old" mathematics programs.

A different picture for classroom practices can be developed from the analysis of official documents following the generalization of the new math to 5th and 6th graders (Wielewski & Matos, 2009). At the end of 1969, one year after the beginning of the new program, the Ministry of Education produced a long memo (Ofício-circular nº 4 116, 1969) with a set of recommendations, from which we may infer some classroom practices. In a stern tone (contrasting with earlier memos, or with official documents issued by the Ministry), teachers were advised to reduce the time devoted to set theory. Several pages of the memo contain "correct" (and incorrect) usages of the terminology associated with sets and their properties. Figure 4 reproduces a particularly harsh page of the memo showing examples of crossed-out questions.

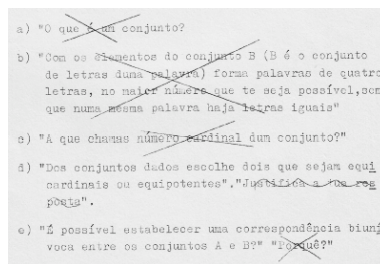


Figure 4. Directions for teachers (Ofício-circular nº 4 116, 1969, p. 9).

Other memos with similar content and attitude followed shortly after this set of recommendations, revealing the difficulties of implementing the program. On the one hand, the didactical use of sets raised many issues related to the linguistic coherence of mathematics that had not been anticipated. On the other hand, it is likely that many teachers felt very uncomfortable with a mathematical content they had not experienced previously, so they just tried to reproduce in class the theoretical approaches of set theory they had been exposed to in in-service short-term courses.

Conclusion

The Modern Mathematics movement affected Portuguese school mathematics culture in two major ways, both at the representational level, and both paralleling international trends. First, there were major recompositions of the representation of what constitutes appropriate mathematical content. Second, mathematics was believed to be a major driver for social and economic development. In the particular case of technical schools, contradictions between instrumental and formative perspectives of mathematics were exposed. With the dimension of school culture that related to practices, changes seem to assume a much narrower facet. However, more research is needed to better understand the ways in which the reform impacted classroom practice.

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